

# Multiphase Transformer

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Multiphase transformers can represent either a 1, 2, or 3-phase transformer. In the corresponding Excel tab, there are columns for declaration of up to 6 connection points (3 for **Winding From** end and 3 for **Winding To** end).

The sending and receiving connection points must be filled in correspondingly:

- For example, if **Winding From/Bus 1** is filled, **Winding To/Bus 1** must be filled as well.
- The unused connection points can be left empty. However, these empty fields must be located immediately after any completed connection point. For example, data in which **Winding From/Bus 2** and **Winding To/Bus 2** fields are filled out but **Winding From/Bus 1** and **Winding To/Bus 1** are empty is not valid and it causes an error.

Finally, the sending and receiving points can be connected to different phases. For example, **Winding From/Bus 1** can be connected to phase A while **Winding To/Bus 1** is connected to phase B.

In summary:

To represent a ...	Fill these columns to add connection points...
single-phase transformer	Only <b>Winding From/Bus 1</b> and <b>Winding To/Bus 1</b> columns.
two-phase transformer	The <b>Winding From/Bus1</b> and <b>Winding From/Bus 2</b> as well as <b>Winding To/Bus 1</b> and <b>Winding To/Bus 2</b> .
three-phase transformer	The <b>Winding From/Bus1</b> , <b>Winding From/Bus 2</b> , <b>Winding From/Bus 3</b> as well as <b>Winding To/Bus 1</b> , <b>Winding To/Bus 2</b> and <b>Winding To/Bus 3</b> .

Multiphase 2W Transformer

Symbol		Description	Unit
<b>ID</b>		Transformer name	unique name
<b>Status</b>		Connect/Disconnect status	Initial value 1 (0 for disconnected)
<b>Number of Phases</b>		Phase count in use	1, 2, or 3
<b>Winding From</b>	<b>Bus1</b>	Primary side: Bus 1	a unique name
	<b>Bus2</b>	Primary side: Bus 2	a unique name
	<b>Bus3</b>	Primary side: Bus 3	a unique name
	<b>V (kV)</b>	Primary winding nominal voltage (phase-to-phase)	kV
	<b>S_base (kVA)</b>	Nominal power in primary side	kVA
	<b>Conn. type (*)</b>	Primary winding connection type	'wye' and 'delta'
<b>Winding To</b>	<b>Bus1</b>	Secondary side: Bus 1	a unique name
	<b>Bus2</b>	Secondary side: Bus 2	a unique name
	<b>Bus3</b>	Secondary side: Bus 3	a unique name
	<b>V (kV)</b>	Secondary winding nominal voltage (phase-to-phase)	kV
	<b>S_base (kVA)</b>	Nominal power in secondary side	NOT APPLICABLE
	<b>Conn. type (*)</b>	Secondary winding connection type	'wye' and 'delta'

Tap 1	Initial tap position: winding 1	Integer between Lowest and Highest Tap
Tap 2	Initial tap position: winding 2	
Tap 3	Initial tap position: winding 3	
Lowest Tap	The lowest tap position	Integer value
Highest Tap	The highest tap position	Integer value
Min Range (%)	Max voltage buck	0 < value < 100
Max Range (%)	Max voltage boost	value > 0
X (pu)	Total reactance	p.u.
Rw1 (p.u.)	Primary winding resistance	p.u.
Rw2 (p.u.)	Secondary winding resistance	p.u.

#### Multiphase 2W Transformer with Mutual Impedance

Symbol	Description	Unit	
ID	Transformer name	a unique name	
Status	Connect/Disconnect status	Initial value 1 (0 for disconnected)	
Number of Phases	Phase count in use	1, 2, or 3	
Winding From	Bus1	Primary side: Bus 1	a unique name
	Bus2	Primary side: Bus 2	a unique name
	Bus3	Primary side: Bus 3	a unique name
	V (kV)	Primary winding nominal voltage (phase-to-phase)	kV
	S_base (kVA)	Nominal power in primary side	kVA
	Conn. type (*)	Primary winding connection type	'wye' and 'delta'
Winding To	Bus1	Secondary side: Bus 1	a unique name
	Bus2	Secondary side: Bus 2	a unique name
	Bus3	Secondary side: Bus 3	a unique name
	V (kV)	Secondary winding nominal voltage (phase-to-phase)	kV
	S_base (kVA)	Nominal power in secondary side	NOT APPLICABLE
	Conn. type (*)	Secondary winding connection type	'wye' and 'delta'
Tap 1	Initial tap position: winding 1	Integer between Lowest and Highest Tap	
Tap 2	Initial tap position: winding 2		
Tap 3	Initial tap position: winding 3		
Lowest Tap	The lowest tap position	Integer value	
Highest Tap	The highest tap position	Integer value	
Min Range (%)	Max voltage buck	0 < value < 100	
Max Range (%)	Max voltage boost	value > 0	
Z0 leakage (pu)	Zero-sequence impedance	transformer p.u.	

<b>Z1 leakage (pu)</b>	Positive-sequence impedance	transformer p.u.
<b>X0/R0</b>	Zero-sequence reactance to resistance ratio	ratio
<b>X1/R1</b>	Positive-sequence reactance to resistance ratio	ratio
<b>No Load Loss (kW)</b>	No-load power loss	NOT APPLICABLE

**Note:** (\*) Four types of winding configurations are supported: 'DD0', 'YgYg0', 'DYg1', 'YgD1'.

#### Available I/O Pins

No	Pin Description	Pin Type	Value/Unit	Instruction
1	Get sending end current magnitude of wire $j$	O	A (RMS)	<b>transformerID/lmagFromj</b> where $j$ is 1, 2 or 3
2	Get receiving end current magnitude of wire $j$	O	A (RMS)	<b>transformerID/lmagToj</b> where $j$ is 1, 2 or 3
3	Get sending end current angle of wire $j$	O	Degree	<b>transformerID/langFromj</b> where $j$ is 1, 2 or 3
4	Get receiving end current angle of wire $j$	O	Degree	<b>transformerID/langToj</b> where $j$ is 1, 2 or 3
5	Set/Get tap position	I/O	Integer between [min_tap, max_tap]	<b>transformerID/tap_j</b> where $j$ is 1, 2 or 3

#### Model Equations

This multiphase transformer is modeled based on the primitive nodal admittance matrix **Yprim** [1],[2].

$\mathbf{Y_{prim}} = \mathbf{A N B Z_B^{-1} B^T N^T A^T}$  matrix dimension:  $np*m \times np*m$ ,  $np$  = number of phases,  $m$ = number of windings

$\mathbf{Y_1} = \mathbf{B Z_B^{-1} B^T}$  ;  $\mathbf{Y_w} = \mathbf{N Y_1 N^T}$  ;  $\mathbf{Y_{prim}} = \mathbf{A Y_w A^T}$

$\mathbf{Y_1}$  is the ground-referenced nodal admittance matrix on a 1 volt base. Matrix dimension:  $np*m \times np*m$

**N** is the incidence matrix whose non-zero elements are the inverse of the numbers of turns in the windings. This matrix represents the effect of the ideal transformers shown to obtain actual windings voltages. Matrix dimension:  $2*np*m \times np*m$

**B** is the incidence matrix whose elements are either 1,-1 or 0. It relates currents in the short circuit reference frame where the first winding is assumed shorted to the currents in the nodal admittance reference frame on a 1 volt base. Matrix dimension:  $np*m \times np$

**A** is the incidence matrix whose non-zero elements are generally either 1 and -1, that relates the winding currents to the actual terminal currents. Matrix dimension:  $nc \times 2*np*m$ ,  $nc$  = number of terminal currents

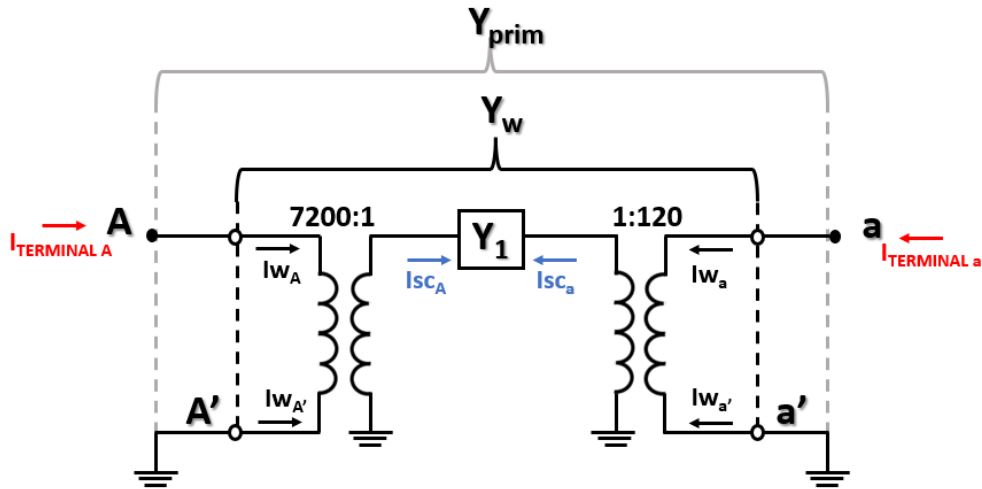
$\mathbf{Z_B}$  is the short circuit impedance matrix. Matrix dimension:  $np*(m-1) \times np*(m-1)$

$\mathbf{Y_w}$  is the winding admittance matrix. Matrix dimension:  $2*np*m \times 2*np*m$

#### Examples

1) A single-phase 2W transformer with the following data: 7.2/0.12 kV, 25 kVA, X = 20%, R=1.1%

In this case  $np = 1$ ,  $m = 2$ .



$Z_B$  in pu =  $0.011+0.02i$ ,  $Z_B$  in 1V base =  $(Z_B \text{ in pu}) \cdot 1^2/25 \text{ kVA} = 4.4e-7 + 8e-7i$ .  $Z_B^{-1} = 527.831e3 - 959.692e3i$

$Y_1 = B Z_B^{-1} B^T$ ;  $B$  is a matrix  $[np \cdot m = 2 \times np = 1]$

$B =$

1
-1

$Y_1 =$

$527.831e3 - 959.692e3i$	$-527.831e3+959.692e3i$
$-527.831e3+959.692e3i$	$527.831e3 - 959.692e3i$

$N$  is a matrix  $[2 \cdot np \cdot m = 4 \times np \cdot m = 2]$

$N =$

$1/7200$	0
$-1/7200$	0
0	$1/120$
0	$-1/120$

$Y_w = N Y_1 N^T =$

$0.0102-0.0185i$	$-0.0102+0.0185i$	$-0.6109+1.1108i$	$0.6109-1.1108i$
$-0.0102+0.0185i$	$0.0102-0.0185i$	$0.6109-1.1108i$	$-0.6109+1.1108i$
$-0.6109+1.1108i$	$0.6109-1.1108i$	$36.6549-66.6453i$	$-36.6549+66.6453i$
$0.6109-1.1108i$	$-0.6109+1.1108i$	$-36.6549+66.6453i$	$36.6549-66.6453i$

To generate matrix  $A$  is necessary to define the number of terminal currents in the model. In this case there are 2 terminal currents (see the red currents in the figure above) so  $nc=2$  and  $A$  matrix is  $[nc=2 \times 2 \cdot np \cdot m=4]$

$A =$

1	0	0	0
0	0	1	0

Finally the matrix  $Y_{prim}$  is calculated

$Y_{prim} = A Y_w A^T =$

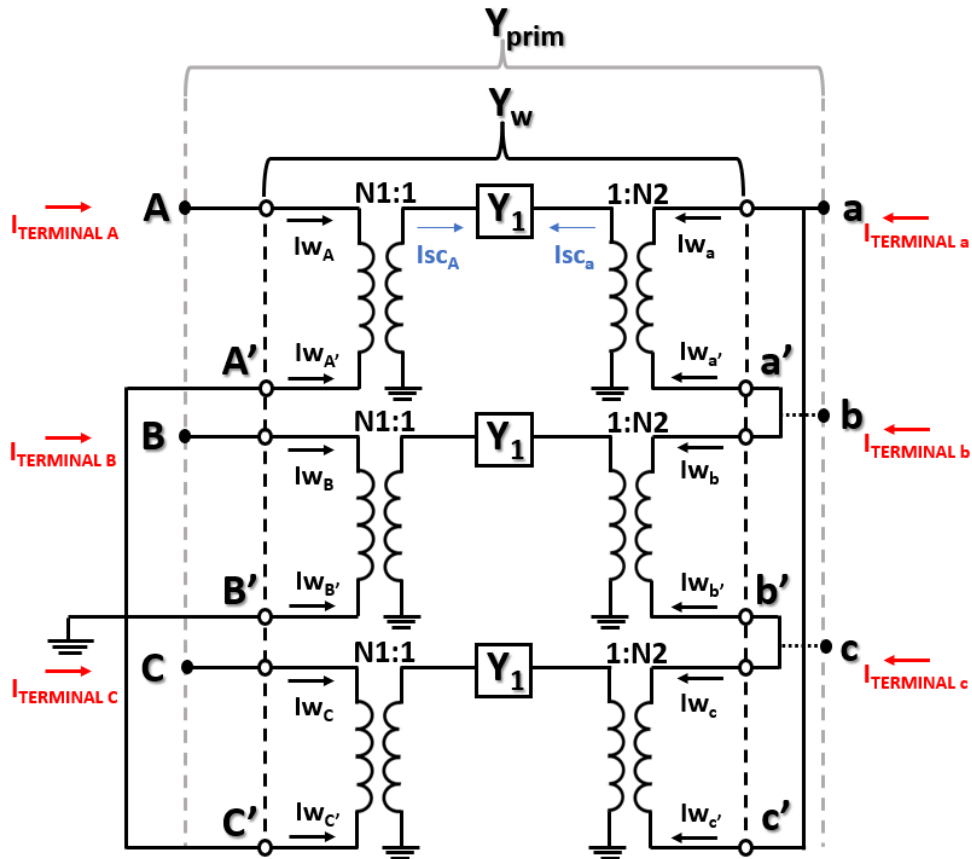
0.0101-0.0185i	-0.6105+1.1100i
-0.6105+1.1100i	36.6007-66.5467i

Below it can be seen how to add this single-phase transformer in the excel file. The total resistance was divided equally between the 2 windings ( $RW1 = RW2 = 0.011 \text{ pu}/2 = 0.0055 \text{ pu}$ ). Note that the voltages must be added as phase to phase voltages even though the model is single-phase (according to the table above)

Multiphase 2W-Transformer		Go to Type List																					
ID	Status	Number of phases	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Tap 1	Tap 2	Tap 3	Lowest Ta	Highest Ta	Min Range	Max Rang	X (pu)	RW1 (pu)	RW2 (pu)	
TRX_B1_B2_a		1 1	B1_a			12.47	25	wye	B2_a		0.208	25	wye			0				0	0.02	0.0055	0.0055
End of Multiphase 2W-Transformer																							

2) A three-phase 2W transformer with the following data: 12.47/0.208 kV (wye/delta), 75 kVA,  $X = 20\%$ ,  $R=1.1\%$

In this case  $np = 3$ ,  $m = 2$ .



$$Z_B \text{ in pu} = 0.011+0.02i, Z_B \text{ in 1V base} = (Z_B \text{ in pu}) * 1^2/75 \text{ kVA} = 1.4667e-7 + 2.6667e-7i. Z_B^{-1} = 158.349e3 - 287.907e3i$$

$$Y_1 = B Z_B^{-1} B^T; B \text{ is a matrix } [np * m = 6 \times np = 3]$$

$B =$

1	0	0
-1	0	0
0	1	0
0	-1	0
0	0	1
0	0	-1

$Y_1 =$

158.349e3 - 287.907e3i	-158.349e3 + 287.907e3i	0	0	0	0
-158.349e3 + 287.907e3i	158.349e3 - 287.907e3i	0	0	0	0
0	0	158.349e3 - 287.907e3i	-158.349e3 + 287.907e3i	0	0
0	0	-158.349e3 + 287.907e3i	158.349e3 - 287.907e3i	0	0
0	0	0	0	158.349e3 - 287.907e3i	-158.349e3 + 287.907e3i
0	0	0	0	-158.349e3 + 287.907e3i	158.349e3 - 287.907e3i

**N** is a matrix [2\*np\*m=12 x np\*m=6]

**N** =

1/12470	0	0	0	0	0
-1/12470	0	0	0	0	0
0	1/(208*sqrt(3))	0	0	0	0
0	-1/(208*sqrt(3))	0	0	0	0
0	0	1/12470	0	0	0
0	0	-1/12470	0	0	0
0	0	0	1/(208*sqrt(3))	0	0
0	0	0	-1/(208*sqrt(3))	0	0
0	0	0	0	1/12470	0
0	0	0	0	-1/12470	0
0	0	0	0	0	1/(208*sqrt(3))
0	0	0	0	0	-1/(208*sqrt(3))

**Y<sub>w</sub> = N Y<sub>1</sub> N<sup>T</sup> =**

0.0102-0.0185i	-0.0102+0.0185i	-0.3525+0.6409i	0.3525-0.6409i	0	0	0	0	0	0	0	0	0
-0.0102+0.0185i	0.0102-0.0185i	0.3525-0.6409i	-0.3525+0.6409i	0	0	0	0	0	0	0	0	0
-0.3525+0.6409i	0.3525-0.6409i	12.2002-22.1823i	-12.2002+22.1823i	0	0	0	0	0	0	0	0	0
0.3525-0.6409i	-0.3525+0.6409i	-12.2002+22.1823i	12.2002-22.1823i	0	0	0	0	0	0	0	0	0
0	0	0	0	0.0102-0.0185i	-0.0102+0.0185i	-0.3525+0.6409i	0.3525-0.6409i	0	0	0	0	0
0	0	0	0	-0.0102+0.0185i	0.0102-0.0185i	0.3525-0.6409i	-0.3525+0.6409i	0	0	0	0	0
0	0	0	0	-0.3525+0.6409i	0.3525-0.6409i	12.2002-22.1823i	-12.2002+22.1823i	0	0	0	0	0
0	0	0	0	0.3525-0.6409i	-0.3525+0.6409i	-12.2002+22.1823i	12.2002-22.1823i	0	0	0	0	0
0	0	0	0	0	0	0	0	0.0102-0.0185i	-0.0102+0.0185i	-0.3525+0.6409i	0.3525-0.6409i	0
0	0	0	0	0	0	0	0	-0.0102+0.0185i	0.0102-0.0185i	0.3525-0.6409i	-0.3525+0.6409i	0
0	0	0	0	0	0	0	0	-0.3525+0.6409i	0.3525-0.6409i	12.2002-22.1823i	-12.2002+22.1823i	0
0	0	0	0	0	0	0	0	0.3525-0.6409i	-0.3525+0.6409i	-12.2002+22.1823i	12.2002-22.1823i	0

To generate matrix **A** is necessary to define the number of terminal currents in the model. In this case there are 6 terminal currents (see figure above) so  $n_c=6$  and **A** matrix is [nc=6 x 2\*np\*m=12]

**A** =

1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	1	0	0	0

$$Y_{prim} = A Y_w A^T =$$

0.0101-0.0185i	0	0	-0.3524+0.6408i	0.3524-0.6408i	0
0	0.0101-0.0185i	0	0	-0.3524+0.6408i	0.3524-0.6408i
0	0	0.0101-0.0185i	0.3524-0.6408i	0	-0.3524+0.6408i
-0.3524+0.6408i	0	0.3524-0.6408i	24.4004-44.3645i	-12.2002+22.1822i	-12.2002+22.1822i
0.3524-0.6408i	-0.3524+0.6408i	0	-12.2002+22.1822i	24.4004-44.3645i	-12.2002+22.1822i
0	0.3524-0.6408i	-0.3524+0.6408i	-12.2002+22.1822i	-12.2002+22.1822i	24.4004-44.3645i

The following image shows how to add this component in the excel file.

Multiphase 2W-Transformer													Go to Type List										
ID	Status	Number of phases	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Tap 1	Tap 2	Tap 3	Lowest Tap	Highest Tap	Min Range	Max Range	X (pu)	RW1 (pu)	RW2 (pu)	
TR1_633_634	1	3	B1_a	B1_b	B1_c	12.47	75	wye	B2_a	B2_b	B2_c	0.208	delta	0	0	0	0	0	0	0	0.02	0.0035	0.0035
End of Multiphase 2W-Transformer																							

### 3) Multiple transformers in the same model

See the **Transformer** page in phasor08\_IEEE13.xls file in demo PHASOR-08.

Multiphase 2W-Transformer													Go to Type List										
ID	Status	Number of phases	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Tap 1	Tap 2	Tap 3	Lowest Tap	Highest Tap	Min Range	Max Range	X (pu)	RW1 (pu)	RW2 (pu)	
TR1_633_634	1	3	B1_a	B1_b	B1_c	12.47	75	wye	B2_a	B2_b	B2_c	0.208	delta	0	0	0	0	0	0	0	0.02	0.0035	0.0035
End of Multiphase 2W-Transformer																							

Positive-Sequence 2W-Transformer													Go to Type List			
ID	Status	From bus	To bus	R (pu)	Xl (pu)	Gmag (pu)	Bmag (pu)	Ratio W1	Ratio W2	Phase Shift (deg)						
End of Positive-Sequence 2W-Transformer																

Positive-Sequence 3W-Transformer													Go to Type List			
ID	Status	Bus1	Bus2	Bus3	R_12 (pu)	Xl_12 (pu)	R_23 (pu)	Xl_23 (pu)	R_31 (pu)	Xl_31 (pu)	Gmag (pu)	Bmag				
End of Positive-Sequence 3W-Transformer																

Multiphase 2W-Transformer													Go to Type List			
ID	Status	Number of phases	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)				
TR1_633_634	1	3	633_a	633_b	633_c	4.16	500	wye	634_a	634_b	634_c	0.48				
TR1_650_651	1	3	650_a	650_b	650_c	4.16	5000	wye	651_a	651_b	651_c	4.16				
End of Multiphase 2W-Transformer																

Multiphase 2W-Transformer with Mutual Impedance													Go to Type List			
ID	Status	Number of phases	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)	Bus1	Bus2	Bus3	V (kV)	S_base (kV Conn. type)				
End of Multiphase 2W-Transformer with Mutual Impedance																

### References

[1] Roger C. Dugan, "A Perspective on Transformer Modeling for Distribution Systems Analysis". 2003 IEEE Power Engineering Society General Meeting. DOI: 10.1109/PES.2003.1267146

[2] Roger C. Dugan and Surya Santoso, "An Example of 3-phase Transformer Modeling for Distribution Systems Analysis". 2003 IEEE PES Transmission and Distribution Conference and Exposition. DOI: 10.1109/TDC.2003.1335084